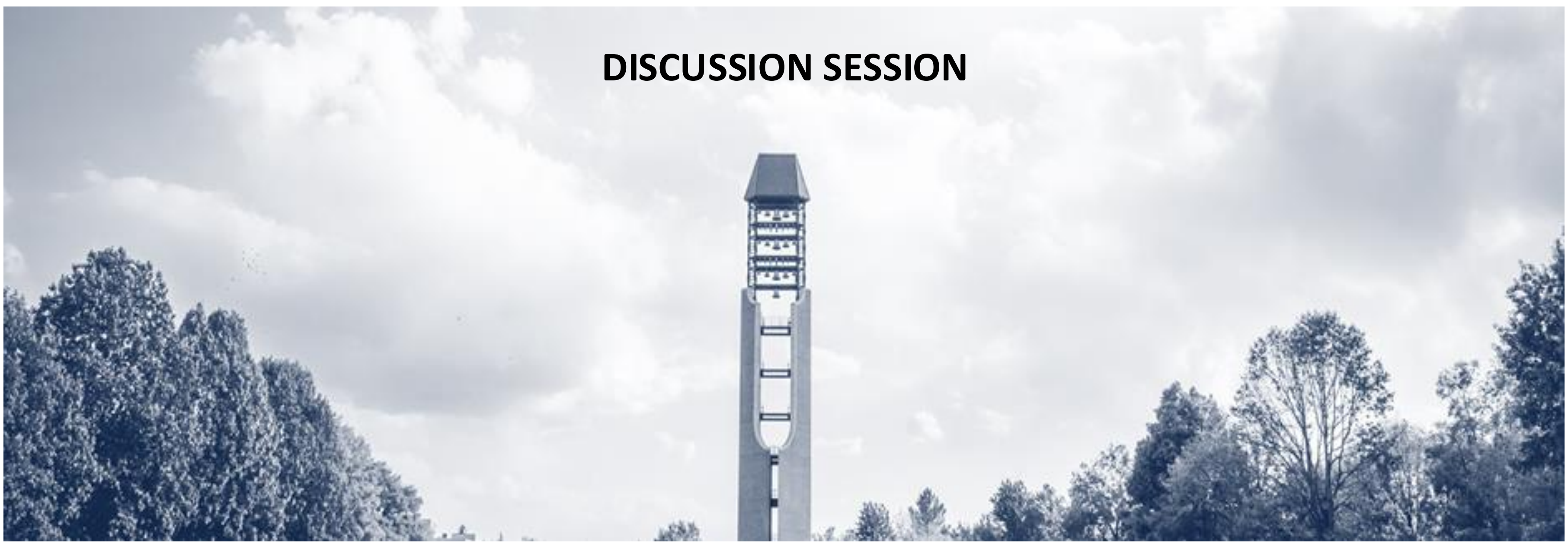
Week 4

ILLINOIS

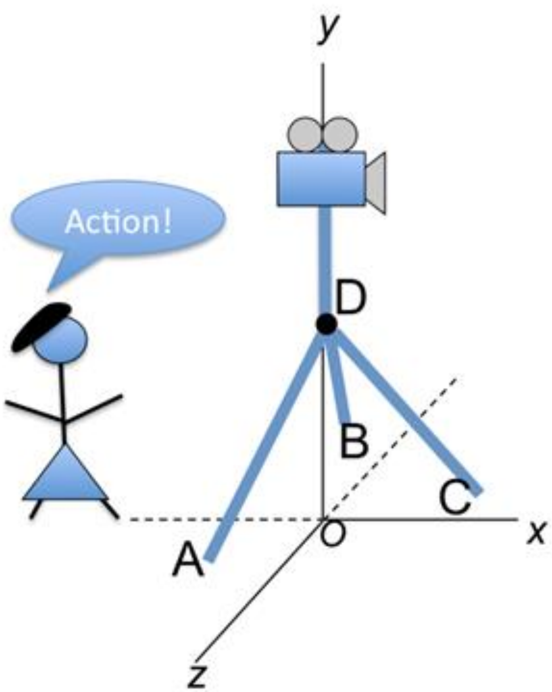
TAM 210/211 Introduction of Statics

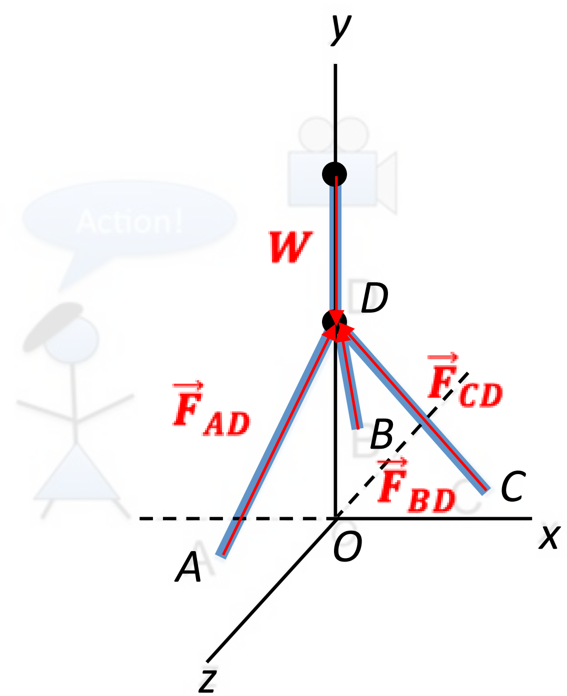


WEEK 4 – Particle Equilibrium

- **Particle equilibrium**
  - The simplest possible case in statics: an object = a particle (a point), all forces = concentrated forces
  - $\Sigma_i^{all}\vec{F_i} = 0 \rightarrow \Sigma_i^{all}F_{ix} = 0,\ \Sigma_i^{all}F_{iy} = 0, \Sigma_i^{all}F_{iz} = 0$

Homework 2A – 3D equilibrium





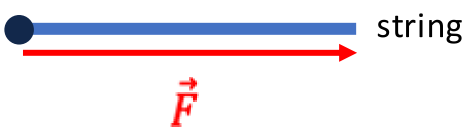
WEEK 4 – Particle Equilibrium

- **How to solve particle equilibrium problems**
  1. Draw a free body diagram
     - Draw external forces on the particle (point)

Unknown force direction?
1. Assume its direction
2. Solve the magnitude of the force F
3. If F > 0, first assumption was right
4. If F < 0, first assumption was wrong → the actual direction is opposite to what you assumed

Exceptions (do not assume the direction)
1. Pre-defined forces: gravity, specified external loads, or hydrostatic pressure
2. Cables, ropes, and strings: tension only (from a point)

     - Decompose forces into x, y, z components



  2. Set up the equilibrium equations
     - $\Sigma_i^{all}\vec{F_i} = 0 \rightarrow \Sigma_i^{all}F_{ix} = 0,\ \Sigma_i^{all}F_{iy} = 0, \Sigma_i^{all}F_{iz} = 0$
  3. Solve the system of equations: linear algebra
     - Define known values and unknown values
     - Convert a system of equations into matrix-vector form

WEEK 4 – Particle Equilibrium

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  3. Solve the system of equations: linear algebra
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$$x + y + z =$$

$$y - 3z =$$

$$2x + 4y - z = 1$$

System of equations

$$\begin{array}{c} \begin{array}{ccc} x & y & \\ \times & \times & \end{array} \\ \underbrace{\begin{bmatrix} 1 & 1 & \\ 0 & 1 & - \\ 2 & 4 & - \end{bmatrix}}_{A} \end{array} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}_{\text{unknown}}} = \underbrace{\begin{bmatrix} 6 \\ 7 \\ 10 \end{bmatrix}}_{\vec{b}}$$

Matrix-vector multiplication

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}_{\text{unknown}}} = \underbrace{\begin{bmatrix} 1 & 1 & \\ 0 & 1 & - \\ 2 & 4 & - \end{bmatrix}^{-1}}_{A^{-1}} \underbrace{\begin{bmatrix} 6 \\ 7 \\ 10 \end{bmatrix}}_{\vec{b}}$$

WEEK 4 – Particle Equilibrium

- **How to solve particle equilibrium problems**
  3. Solve the system of equations: linear algebra
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$$\Sigma_i^{all}\vec{F_i} = \vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$
$$= |\vec{F_1}|\vec{u}_{F_1} + |\vec{F_2}|\vec{u}_{F_2} + |\vec{F_3}|\vec{u}_{F_3}$$
$$= 0$$

Equilibrium equation

$$\Sigma_i^{all}F_{ix} = |\vec{F_1}|u_{F_1,x} + |\vec{F_2}|u_{F_2,x} + |\vec{F_3}|u_{F_3,x} = 0 \qquad \Sigma_i^{all}F_{ix} = 0$$
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System of equations

$$\underbrace{\begin{bmatrix} u_{F_1,x} & u_{F_2,x} & u_{F_3,x} \\ u_{F_1,y} & u_{F_2,y} & u_{F_3,y} \\ u_{F_1,z} & u_{F_2,z} & u_{F_3,z} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} |\vec{F_1}| \\ |\vec{F_2}| \\ |\vec{F_3}| \end{bmatrix}}_{\vec{x}_{\text{unknown}}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\vec{b}}$$

Matrix-vector multiplication

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Solve for variables

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